SEKOLAH BUKIT SION

IGCSE Mock Examination 2021

STUDENT NAME			
EXAMINEE NUMBER		CENTRE ID	

0606 ADDITIONAL MATHEMATICS (PAPER 2)

Year 10

10 April 2021

2 hours

Additional Materials:

- Scientific Calculator
- Ruler

READ THESE INSTRUCTIONS FIRST

Write your name, exam number and grade on all the work you hand in.
Write in dark blue or black pen.
Use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

Score :

1 **(a)** Differentiate $y = (3x^2 - 1)^{-\frac{1}{3}}$ with respect to x. [2]

$$\frac{dy}{dx} = -\frac{1}{\cancel{3}}(3x^2-1)^{-4/3} \cdot \cancel{6}x$$

$$\frac{dy}{dx} = -2x(3x^2-1)^{-4/3}$$

(b) Find the approximate change in y as x increases from $\sqrt{3}$ to $\sqrt{3} + p$, where p is small. [1]

$$\delta y = -2\sqrt{3}\,(8)^{-4/3} \cdot p$$

$$= -2\sqrt{3}\left(\frac{1}{24}\right) \cdot p$$

$$\delta y = \frac{p\sqrt{3}}{-8}$$

(c) Find the equation of the normal to the curve $y = (3x^2 - 1)^{-\frac{1}{3}}$ at the point where $x = \sqrt{3}$. [3]

when $x = \sqrt{3}$, $y = \frac{1}{2}$

$$m_t = \frac{\sqrt{3}}{-8} \qquad m_n = \frac{8}{\sqrt{3}} = \frac{8\sqrt{3}}{3}$$

$$y = \frac{8\sqrt{3}}{3}x + c$$

$$\frac{1}{2} = \frac{8\cancel{\sqrt{3}}}{\cancel{3}}(\cancel{\sqrt{3}}) + c$$

$$\frac{1}{2} = 8 + c \Rightarrow -\frac{15}{2}$$

$$\therefore y = \frac{8}{\sqrt{3}}x - \frac{15}{2}$$

Question	Answer	Marks	AO Element	Notes	Guida
1(a)	**M1** $\frac{\mathrm{d}y}{\mathrm{d}x} = kx\left(3x^2 - 1\right)^{-\frac{4}{3}}$ **A1** $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{3} \times 6x\left(3x^2 - 1\right)^{-\frac{4}{3}}$	**2**			
1(b)	**B1** When $x = \sqrt{3}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{\sqrt{3}}{8}$ $-\frac{\sqrt{3}}{8}p$ or $-0.217p$	**1**		**FT** on *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$ of the form $kx\left(3x^2 - 1\right)^{-\frac{4}{3}}$	
1(c)	**B1** When $x = \sqrt{3}$, $y = \frac{1}{2}$ **M1 A1** Normal: $y - \frac{1}{2} = \frac{8}{\sqrt{3}}\left(x - \sqrt{3}\right)$	**3**		for $y = \frac{1}{2}$ **M1** Dep on M1 in part **(a)**. An equation of the normal using *their* normal gradient, $\sqrt{3}$ and *their* y **A1** allow unsimplified	
					[Total: 6]

2 Without using a calculator, express $\frac{(\sqrt{5}-3)^2}{\sqrt{5}+1}$ in the form of $p\sqrt{5}+q$,
where p and q are integers. [5]

Question	Answer	Marks	AO Element	Notes	Guidan
1	**M1** $(\sqrt{5}-3)^2 = 5+9-2(3)\sqrt{5}$ **M1** $\frac{\textit{their}\ (14-6\sqrt{5})}{\sqrt{5}+1} \times \frac{\sqrt{5}-1}{\sqrt{5}-1}$ **M1** $\frac{\textit{their}\ (14\sqrt{5}-30-14+6\sqrt{5})}{5-1}$ **A1** $5\sqrt{5}-11$	4		Attempts to rationalise or forms a pair of simultaneous equations e.g. $5p+q=14,\ p+q=-6$ multiplies out; numerator must have at least 3 terms; condone one sign error in numerator; denominator may be 4 or $5-\sqrt{5}+\sqrt{5}-1$ or solves *their* simultaneous equations to find one unknown or $p=5,\ q=-11$	
					[Total: 4]

Close

$$\frac{(\sqrt{5}-3)^2}{\sqrt{5}+1}$$

$$\frac{5-6\sqrt{5}+9}{\sqrt{5}+1}$$

$$\frac{14-6\sqrt{5}}{\sqrt{5}+1} \cdot \frac{\sqrt{5}-1}{\sqrt{5}-1}$$

$$\frac{14\sqrt{5}-14-30+6\sqrt{5}}{4}$$

$$\frac{20\sqrt{5}-44}{4}$$

$$5\sqrt{5}-11$$

3 Find the values of k for which the equation $(k-1)x^2 + kx - k = 0$ has real distinct roots. [4]

Question	Answer	Marks	AO Element	Notes	Guidance
1	**B1** $k^2 - 4(k-1)(-k)$ oe **M1** $k(5k-4)$ **A1** Correct critical values 0, 0.8 oe **A1** $k < 0,\ k > 0.8$ oe	**4**		Mark scheme **FT** *their* critical values provided *their* $ak^2 + bk + c > 0$ has positive a and there are 2 values; mark final answer If **B1 M0** allow **SC1** for a final answer of $k > 0.8$ oe	
[Total: 4]					

4 Given that $4x^4 - 12x^3 - b^2x^2 - 7bx - 2$ is exactly divisible by $2x + b$,

(i) show that $3b^3 + 7b^2 - 4 = 0$, [2]

$2x + b = 0$
$2x = -b$
$x = -\frac{b}{2}$

$4\left(-\frac{b}{2}\right)^4 - 12\left(-\frac{b}{2}\right)^3 - b^2\left(-\frac{b}{2}\right)^2 - 7b\left(-\frac{b}{2}\right) - 2 = 0$

$\cancel{4}\left(\frac{b^4}{16_4}\right) + \cancel{12}^3\left(\frac{b^3}{8_2}\right) - b^2\left(\frac{b^2}{4}\right) + \frac{7b^2}{2} - 2 = 0$

$2\times\left(\frac{\cancel{b^4}}{\cancel{4}} + \frac{3}{2}b^3 - \frac{\cancel{b^4}}{\cancel{4}} + \frac{7b^2}{2} - 2 = 0\right)\times 2$

$3b^3 + 7b^2 - 4 = 0$ shown

(ii) find the possible values of b. [5]

$$\begin{array}{r|rrrr} -2 & 3 & 7 & 0 & -4 \\ & & -6 & -2 & 4 \\ \hline & 3 & 1 & -2 & 0 \end{array}$$

$\Rightarrow 3x^2 + x - 2$
$(3x-2)(x+1)$

$(x+2)(3x-2)(x+1) = 0$

$x = -2, \frac{2}{3}, -1$ #

5 The points A and B have coordinates $(p, 3)$ and $(1, 4)$ respectively and the line L has equation

$3x + y = 2$.

(a) Given that the gradient of AB is $\frac{1}{3}$, find the value of p. [2]
(b) Show that L is the perpendicular bisector of AB. [3]
(c) Given that $C(q, -10)$ lies on L, find the value of q. [1]
(d) Find the area of triangle ABC. [2]

Question	Answer	Marks	AO Element	Notes	Guidan
1(a)	**M1** $\frac{4-3}{1-p} = \frac{1}{3}$ oe **A1** -2	2		**ALT** uses $y = mx + c$ with A and B as far as an equation in p only	
1(b)	Method 1 **B1** Finds midpoint AB $\left(\frac{their\ p+1}{2}, \frac{3+4}{2}\right)$ **B1** Verifies $(-0.5, 3.5)$ is on L **B1** $y = -3x + 2$ therefore $m = -3$ oe **and** $\frac{1}{3} \times -3 = -1$ Method 2 **B1** Finds midpoint AB $\left(\frac{their\ p+1}{2}, \frac{3+4}{2}\right)$ **B1** $\frac{1}{3} \times -3 = 1$ oe **B1** $y - 3.5 = -3(x + 0.5)$ **and** completion to $y = -3x + 2$	3		**FT** *their p* **FT** *their p* Mark scheme	
1(c)	**B1** $q = 4$	1			

Close

(d)

$$A = \frac{1}{2}\left|\begin{matrix} -2 & 1 & 4 & -2 \\ 3 & 4 & -10 & 3 \end{matrix}\right|$$

$$A = \frac{1}{2}\left|(-8 - 10 + 12) - (3 + 16 + 20)\right|$$

$$= \frac{1}{2}\left|(-45)\right| = \underline{22.5 \text{ sq units}}$$

6 Show that

(a) $$\frac{\operatorname{cosec}\theta - \cot\theta}{\sin\theta} = \frac{1}{1+\cos\theta}.$$ [4]

Question	Answer	Marks	AO Element	Notes	Guidan
1(a)	**M2** $\frac{1}{\sin\theta}\left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)$ **M1** $\frac{1-\cos\theta}{1-\cos^2\theta}$ **A1** $\frac{1-\cos\theta}{(1-\cos\theta)(1+\cos\theta)} = \frac{1}{1+\cos\theta}$	4		**M1** for either $\frac{\operatorname{cosec}\theta - \cot\theta}{\sin\theta} = \frac{1}{\sin\theta}\left(\operatorname{cosec}\theta - \frac{\cos\theta}{\sin\theta}\right)$ or $\frac{\operatorname{cosec}\theta - \cot\theta}{\sin\theta} = \frac{1}{\sin\theta}\left(\frac{1}{\sin\theta} - \cot\theta\right)$	

Close

(b) $(1+\sec\theta)(\operatorname{cosec}\theta - \cot\theta) = \tan\theta.$ [4]

4	[4]	$\left(1+\frac{1}{\cos\theta}\right)\left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right) \Rightarrow \frac{1-\cos^2\theta}{\cos\theta\sin\theta}$ $1-\cos^2\theta \equiv \sin^2\theta$ $\quad \frac{\sin^2\theta}{\cos\theta\sin\theta} \to \tan\theta$ Must be useful use of Pythagoras	M1 M1 B1 A1

7 Solve $6\sin^2 x - 13\cos x = 1$ for $0° \le x \le 360°$. [4]

Question	Answer	Marks	AO Element	Notes	Guidance
1	**B1** $6\left(1-\cos^2 x\right)-13\cos x = 1$ oe **M1** Solves or factorises *their* 3-term quadratic **A2** 70.5 and 289.5	**4**		with no extras in range **A1** for either, ignoring extras in range	
					[Total: 4]

8 Solve.

(a) $1 + \lg(8 - x) = \lg(3x + 2)$ [4]

$\lg 10 + \lg(8-x) = \lg(3x+2)$

$\cancel{\lg}\ 10(8-x) = \cancel{\lg}\ (3x+2)$

$80 - 10x = 3x + 2$

$78 = 13x$

$\underline{x = 6}$

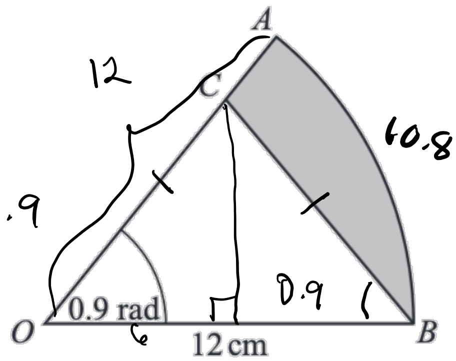
(b) $\log_4(3y^2 - 10) = 2\log_4 y - 1) + \frac{1}{2}$ [5]

Question	Answer	Marks	AO Element	Notes	Guidan
1	Method 1	5			
	B1 $\log_4\left(3y^2 - 10\right) = \log_4 (y-1)^2 + \frac{1}{2}$			use of power rule	
	B1 $\log_4 \frac{3y^2 - 10}{(y-1)^2} = \frac{1}{2}$			**DepB1** for use of division rule	
	B1 $\frac{3y^2 - 10}{(y-1)^2} = 2$			for $\frac{1}{2} = \log_4 2$	
	M1 $y^2 + 4y - 12 = 0$			**Dep** on first two B marks simplification to a three term quadratic.	
	A1 $y = 2$ only				
	Method 2				
	B1 $\log_4\left(3y^2 - 10\right) = \log_4 (y-1)^2 + \frac{1}{2}$			use of power rule	
	B1 $\log_4\left(3y^2 - 10\right) = \log_4 (y-1)^2 + \log_4 2$			for $\log_4 2$	
	B1 $3y^2 - 10 = 2(y-1)^2$			**Dep** on first B1 use of the multiplication rule	
	M1 $y^2 + 4y - 12 = 0$			**Dep** on first and third B marks. simplification to a 3 term quadratic	
	A1 $y = 2$ only				
					[Total: 5]

9

$$\frac{12}{\sin 76.48} = \frac{OC}{\sin 0.9}$$

$OC = BC = 9.65$



$$\cos 0.9 = \frac{6}{OC}$$

$$OC = \frac{6}{\cos 0.9}$$

The diagram shows a sector, *AOB*, of a circle centre *O*, radius 12 cm.
Angle *AOB* = 0.9 radians.
The point *C* lies on *OA* such that *OC* = *CB*.

(a) Show that *OC* = 9.65 cm correct to 3 significant figures. [2]

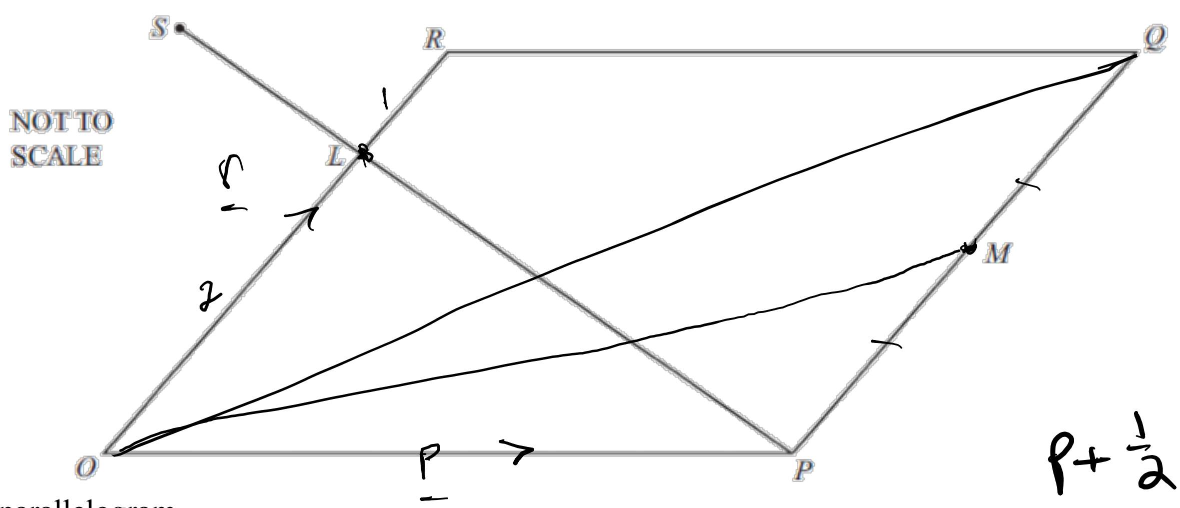
(b) Find the perimeter of the shaded region. [3]

(c) Calculate the area of the shaded region. [3]

Question	Answer	Marks	AO Element	Notes	Guidance
1(a)	$\cos 0.9 = \dfrac{6}{OC}$ or $\dfrac{OC}{\sin 0.9} = \dfrac{12}{\sin(\pi - 1.8)}$	M1		for correct use of cosine, sine rule, cosine rule or any other valid method	
	$OC = \dfrac{6}{\cos 0.9} = 9.652...$ or $OC = \dfrac{12 \sin 0.9}{\sin(\pi - 1.8)} = 9.652...$	A1		for manipulating correctly to *OC* = 9.652(35…) Must have 4th figure (or more) for rounding	
1(b)	0.9×12	B1		for arc length	
	Perimeter = $(0.9\times12) + 9.652 + (12 - 9.652)$	M1		for attempt to add the correct lengths	
	= 22.8	A1			

Question	Answer	Marks	AO Element	Notes	Guidance
1(c)	Area = $\left(\frac{1}{2} \times 12^2 \times 0.9\right) - \left(\frac{1}{2} \times 9.652^2 \sin(\pi - 1.8)\right)$	**B2**		**B1** for area of sector, allow unsimplified **B1** for area of isosceles triangle $\frac{1}{2}(9.65(2\ldots))^2 \sin(\pi - 1.8)$ or $\frac{1}{2}(12 \times 6 \tan 0.9)$ or $\frac{1}{2}(12 \times 9.65(2\ldots) \times \sin 0.9)$, allow unsimplified.	
	64.8 – 45.36 = 19.4 to 19.5 Alternative solution: $\frac{1}{2}(12 - 9.652) \times 9.652 \times \sin 1.8$ (B1) $\frac{1}{2} 12^2 (0.9 - \sin 0.9)$ (B1) 11.04 + 8.40 Area = 19.4 to 19.5 (B1)	**B1**		for answer in range 19.4 to 19.5 **B1** for area of triangle *ACB*, unsimplified **B1** for area of segment, unsimplified **B1** for answer in range 19.4 to 19.5	
					[Total: 8]

10



$p + \frac{1}{2}r$

OPQR is a parallelogram.
O is the origin. $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OR} = \mathbf{r}$.
M is the midpoint of *PQ* and *L* is on *OR* such that *OL* : *LR* = 2 : 1.

The line *PL* is extended to the point *S*.

(a) Find, in terms of **p** and **r,** in their simplest forms,

(i) $\overrightarrow{OQ}$

Answer: $p + r$ .... [1]

(ii) $\overrightarrow{PR}$,

Answer: $-p + r$ .... [1]

(iii) $\overrightarrow{PL}$,

Answer: $-p + \frac{2}{3}r$ .... [2]

(iv) The position vector of *M*.

Answer: $p + \frac{1}{2}r$ .... [1]

(b) *PLS* is a straight line and $PS = \frac{3}{2}PL$.
Find, in terms of **p** and/or **r**, in their simplest forms,

(i) $\overrightarrow{PS}$ $\quad \overrightarrow{PS} = \frac{3}{2}(-p + \frac{2}{3}r)$

Answer: $-\frac{3}{2}p + r$ .... [1]

(ii) $\overrightarrow{QS}$ $\quad \overrightarrow{QS} = \overrightarrow{QP} + \overrightarrow{PS}$

$= \cancel{r} + -\frac{3}{2}p \cancel{+r}$

Answer: $p = -\frac{3}{2}$ .... [2]

11 CHOOSE/ANSWER ONLY 1

EITHER

It is given that $y = (x^2 + 1)(2x - 3)^{\frac{1}{2}}$.

(a) Show that $\frac{dy}{dx} = \frac{Px^2 + Qx + 1}{(2x-3)^{\frac{1}{2}}}$, where P and Q are integers. [5]

Question	Answer	Marks	AO Element	Notes	Guidan
1(a)	**M1 B1 A1** $\frac{dy}{dx} = (x^2+1)(2x-3)^{-\frac{1}{2}}$ **M1** $=(2x-3)^{-\frac{1}{2}}\left(x^2+1+2x(2x-3)\right)$ **A1** $= \frac{5x^2-6x+1}{(2x-3)^{\frac{1}{2}}}$	5		**M1** for differentiation of a product **B1** for $\frac{d}{dx}(2x-3)^{\frac{1}{2}} = \frac{1}{2}\times 2(2x-3)^{-\frac{1}{2}}$ oe **A1** all else correct i.e. $\frac{dy}{dx} = (x^2+1)f(x) + 2x(2x-3)^{\frac{1}{2}}$ correctly taking out a factor of $(2x-3)^{-\frac{1}{2}}$ or correctly using $(2x-3)^{\frac{1}{2}}$ as denominator	

Close

Mark scheme

(b) Hence find the equation of the normal to the curve $y = (x^2 + 1)(2x - 3)^{\frac{1}{2}}$ at the point $x = 2$, giving your answer in the form $ax + by + c = 0$, where a, b, and c are integers. [4]

Question	Answer	Marks	AO Element	Notes	Guidan
1(b)	**B1** When $x = 2$, $y = 5$ **M1** $\frac{dy}{dx} = 9$, so gradient of normal $= -\frac{1}{9}$ **M1** Equation of normal $y - 5 = -\frac{1}{9}(x-2)$ **A1** $x + 9y - 47 = 0$ or $-x - 9y + 47 = 0$	4		substitution to obtain gradient and correct method for gradient of normal **DepM1** for equation of normal Must be in this form	
					[Total: 9]

Close

OR

A closed cylinder has base radius r, height h and volume V. It is given that the total surface area of the cylinder is 600π and that V, r and h can vary.

(a) Show that $V = 300\pi r - \pi r^3$. [3]
(b) Find the stationary value of V and determine its nature. [5]

Question	Answer	Marks	AO Element	Notes	Guidance
1(a)	Method 1 **B1** $600\pi = 2\pi r^2 + 2\pi rh$ **M1** $h = \dfrac{600\pi - 2\pi r^2}{2\pi r}$ **A1** $V = \pi r^2 h$ $V = \pi r^2\left(\dfrac{600\pi - 2\pi r^2}{2\pi r}\right)$ $V = \pi r^2\left(\dfrac{300}{r} - r\right)$ $V = 300\pi r - \pi r^3$ Method 2 **B1** $600\pi = 2\pi r^2 + 2\pi rh$ **M1** $600\pi r = 2\pi r^3 + 2\pi r^2 h$ **A1** $\dfrac{600\pi r - 2\pi r^3}{2} = \pi r^2 h$ $V = \pi r^2 h$ $V = 300\pi r - \pi r^3$	3		making h subject from a two term expression for SA. correct substitution and manipulation to obtain given answer multiplying both sides by r correct manipulation to obtain $\pi r^2 h$	

Close

(b)

$V = 300\pi r - \pi r^3$
$V' = 300\pi - 3\pi r^2 = 0$ ✓
$300\cancel{\pi} = 3\cancel{\pi} r^2$
$\cancel{3}r^2 = \cancel{300}$
$r^2 = 100$
$r = 10$ ✓

$V = 2000\pi$ ✓

$V'' = -6\pi r$ ✓
@ $r = 10$, $-6\pi(10)$
$V'' = -60\pi$
maximum ✓